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### **A Model of Political Campaign Manipulation\***

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#### **RESUMEN**

Proponemos un modelo espacial multi-dimensional de competencia política en el que la campaña electoral tiene como objetivo influir sobre los pesos que asignan las preferencias de los votantes a los diferentes temas políticos. Las estrategias de la campaña electoral modificarán el voto de aquellos votantes que no se identifiquen con ninguno de los partidos. El análisis del equilibrio del juego electoral que proponemos nos permite comprender mejor las posibilidades que tiene el partido político considerado como perdedor ex-ante de ganar las elecciones cuando hay campaña electoral. Demostramos que el perdedor ex-ante puede acabar ganado las elecciones incluso cuando (1) dispone de menos fondos para la campaña que su contrincante, y (2) no dispone de ventaja en ninguno de los diferentes temas políticos.

**Palabras clave:** Campaña electoral, temas políticos, manipulación de preferencias, votación posicional.

**JEL clasificación:** C 70, D 72.

#### **ABSTRACT**

We propose a multidimensional spatial model of political competition where the advertising campaign aims at influencing the weights that voters' preferences assign to different political issues. The campaign strategies will move the vote of those voters who lack of partisan identification. The equilibrium analysis of the proposed electoral game yields insights into the chances that the ex-ante loser political party has of winning the elections when there is electoral campaign. We show that the ex-ante loser can end up winning the elections even when (1) it has less campaign funds than its opponent and, (2) it has no advantage on any single political issue.

**Keywords:** Election campaign, political issues, preferences manipulation, positional voting.

**JEL classification:** C 70, D 72.

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# 1 Introduction

There are several theories that aim at explaining how political campaign expenditure persuades voters:

(1) Campaign funds influence voters who are uninformed about the parties' political positions. It is assumed that a fixed fraction of the voters are uninformed. The proportion of uninformed voters that a political party obtains is increasing in the campaign expenditure of the political party (see, e.g., Baron, 1994, Grossman and Helpman, 1996).

(2) Campaign funds clarify the political positions of the candidates and alleviates risk averse voters' uncertainty (see, e.g., Austen-Smith, 1987).

(3) Campaign funds increase the probability of winning the elections. It is assumed that there is an "electoral production function" which is positively affected by campaign expenditure (see, e.g., Friedman, 1958, Brams and Davis, 1973, Snyder, 1989).<sup>1</sup>

While explanations (1) and (2) relate campaign activities to information acquisition, explanation (3) provides a more general setting where campaign activities work as an input to produce votes. None of these explanations, however, describes how political campaigns may affect voters' preferences. Thus, in (1) and (3) voters' preferences are not described, and in (2) campaign expenditure just reduces voters' uncertainty.<sup>2</sup>

In this paper we aim at modelizing one important aspect which has not being so far considered: how political campaigns may affect voters' preferences. Our proposal does not go through those campaign activities which aim at providing information. It however focus on campaign activities which aim at persuading voters by means of distorting voters' taste. Thus, our proposal resembles, to a certain extend, the effect that the advertising activities have on market economies (see for instance Dixit and Norman 1978).

The model we use is similar to one proposed by Riker and Ordeshook (1973), but we extend it to allow for the effect of campaign expenditure affecting voters' preferences. In particular, we consider a multidimensional spacial model of political competition between two political parties. Parties' political positions are common knowledge, and are described by the policies that they support on each of the political dimensions. Parties aim at ob-

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<sup>1</sup>As pointed out by Snyder (1989), a particular instance of an "electoral production function" is provided by the rent-seeking literature. See, for instance Tullock (1981).

<sup>2</sup>In (1), the informed voters have well-defined preferences on the political space while the non-informed voters cannot be described by an ideal policy.

taining a majority to implement their policy. Voters are represented by their own ideal policies on each political issue, and they prefer the political party which is closest to their ideal policy.

From the empirical evidence (see Laver and Hunt, 1992, Budge, 1993, Riker, 1993, and Petrocik, 1996) and from recent political campaigns in modern democracies, we deduce that competition of political parties is increasingly based on political issues. Indeed, we often observe that the political parties use the political campaigns to promote those issues from which they can capture a greater amount of votes. We therefore hypothesize in this paper that campaign expenditure may affect the relative intensity that the voters assign to some issues over others. Thus, in our model, parties' strategies consist on allocating campaign funds among the different political issues. In this way, the political parties choose the emphasis that they want to make on each of the political issues.

The electorate can be divided into two groups: partisan voters and issue voters. Whereas partisan voters have an ideal policy on each of the political issues which mostly coincides with one of the political parties, issue voters prefer one of the parties in some of the political issues and the other political party in other issues. Thus, the parties' strategies persuade the issue voters to vote for one political party or the other. In elections where the set of partisan voters attached to a political party is not majoritarian, the set of issue voters becomes crucial to determine what political party wins the elections.

Based on the empirical evidence Riker (1993) and Petrocik (1996) provide some ideas on how the political parties compete in political issues. From the analysis of the national campaign of the U.S. for the ratification of the Constitution, Riker (1993) argues that, (i) when one party has a clear-cut advantage on an issue it regularly emphasizes that issue while the other party abandons it (dominance principle), and (ii) when neither side has a clear advantage on an issue, both abandon it (dispersion principle). In a similar vein, from the analysis of the presidential elections of the U.S. between 1960 and 1992, Petrocik (1996) provides the idea of "issue ownership", which is based on the perception that voters have on how a political party handles certain political issues (or political problems). A party that is viewed as better qualified to handle an issue is said to have the ownership of that issue. Thus, it is expected that candidates emphasize issues on which they are advantaged.

None of the above mentioned authors, however, develop a formal theoretical model. Thus, with this paper we complement their analysis by means of

providing a theoretical model which tries to capture all the basic features of parties' competition in political issues.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 studies general conditions for equilibrium. Section 4 studies to what extent the elections result is affected by the the parties' pre-electoral advantage, the parties' advantage on single issues, and the parties' difference in campaign funds. Section 5 provides the conclusions.

## 2 The model

There is a society with a finite set of voters  $N = \{1, \dots, n\}$  which shall select by popular elections a representative to serve in the legislature. There are two political parties  $A$  and  $B$ , that compete for winning a majority of the votes by spending campaign resources. There is a finite set of political issues  $M = \{1, \dots, m\}$ . A political issue describes an objective problem of a society such as unemployment, pensions, migration, terrorism, etc.

### The political parties

Each party  $j \in \{A, B\}$  has a fixed and known **political position**  $x_j = (x_{j1}, \dots, x_{jm}) \in [0, 1]^m$ , where  $x_{jr} \in [0, 1]$  is the political position of party  $j$  on issue  $r \in M$ , and is endowed with some fixed **campaign funds**  $\bar{c}_j > 0$ .<sup>3</sup>

Campaign funds are devoted to advertising campaign and each political party emphasizes those issues which can persuade a greater amount of voters. We define a **campaign strategy** of party  $j$  as a vector  $c_j = (c_{j1}, \dots, c_{jm}) \in C_j = \left\{ \mathbb{R}_+^m : \sum_{r=1}^m c_{jr} \leq \bar{c}_j \right\}$ , which indicates how the party allocates its funds among the different issues.<sup>4</sup> Let  $c = (c_A, c_B) \in C_A \times C_B = C$  denote a profile of campaign strategies. For each  $c \in C$  and each  $r \in M$ , let  $c_r = c_{Ar} + c_{Br}$  be the total funds spent on issue  $r$ .

### The voters

Each voter  $i \in N$  has a fixed and known **ideal political position**  $\pi_i = (\pi_{i1}, \dots, \pi_{im}) \in [0, 1]^m$  where  $\pi_{ir} \in [0, 1]$  is the ideal political position of voter

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<sup>3</sup>Since parties' political positions are fixed, we can assume that each party is aligned with a set of interest groups which provides its campaign funds.

<sup>4</sup>The campaign strategy  $c_j$  can be also interpreted as the time that party  $j$  devotes to advertise each political issue.

$i$  on issue  $r \in M$ . Voters' ideal political positions are distributed according to some given **distribution of voters** on  $[0, 1]^m$ .

Each voter prefers the political party who is closest to his ideal political position. Besides that, campaign strategies also have an influence on voters' preferences. Thus, one of the crucial assumptions of this model is that the intensity of the voters' preferences over each issue  $r$  depends on the campaign expenditure on that issue,  $c_r$ . In particular, the preferences of each voter  $i$  over the political parties are represented by the following utility function:

$$u_i(j, c) = - \sum_{r=1}^m \alpha_r(c_r) [x_{jr} - \pi_{ir}]^2 \quad (1)$$

where, for each  $r \in M$ ,  $\alpha_r(\cdot)$  is a twice continuously differentiable function of the campaign expenditure on issue  $r$  that indicates the weight that each voter assigns to that issue. We will refer to  $\alpha_r(\cdot)$  as the **influence function** on issue  $r$ . We assume that  $\alpha_r(0) > 0$  and  $\frac{\partial \alpha_r(c_r)}{\partial c_r} > 0$ .

Note that we have made the simplifying assumption that all voters are equally influenced by the campaign expenditure, i.e., for any issue  $r$ , the influence function  $\alpha_r(\cdot)$  does not vary among voters. This assumption can be justified on the basis that all voters have an equal access to advertising activities. Riker and Ordeshook (1973), pointed out that there is one relaxation that is generally permitted which consists on considering that there exists some average level of concern for each of the political issues.<sup>5</sup>

Figure 1 illustrates an example of voters' indifference curves for the case of two political issues. The solid curves represent the indifference curves when there is no campaign expenditure. Expending campaign funds can vary the relative importance that voters assign to each issue. Thus, the narrow dotted curves represent the indifference curves when campaign expenditure makes issue 1 more relevant, while the wide dotted curves represent the indifference curves when campaign expenditure makes issue 2 more relevant.

[INSERT FIGURE 1]

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<sup>5</sup>Note also that, while the campaign expenditure determines the intensity of voters' preferences over issues, it has no influence on their ideal political positions. This restriction is probably the smallest step one could take to analyze the effect of campaign expenditure when there are several issues (and it is still reasonable in many settings).



Given any profile of campaign strategies  $c \in C$ , voter  $i$  casts his ballot for party  $j \neq k$  when  $u_i(j, c) > u_i(k, c)$ . We suppose that those voters who are indifferent between the two parties abstain from voting. We denote  $V_j(c)$  to the set of voters that vote for party  $j$  under  $c$ , i.e.,  $V_j(c) = \{i \in N : u_i(j, c) > u_i(k, c)\}$ .

### The campaign game

Given any profile of campaign strategies  $c \in C$ , let  $\#V_j(c)$  be the number of votes that party  $j$  obtains in the elections. If  $\#V_j(c) > \#V_k(c)$  party  $j$  wins the elections. For simplicity, we assume that if parties are involved in a tie, then party  $A$  will govern.

Political parties aim at winning the elections in order to implement their political position. Preferences of each party  $j \neq k$  are represented by the following utility function:

$$\begin{aligned} w_j(c) &= \begin{cases} 1; & \text{if } \#V_j(c) > \#V_k(c) \\ 0; & \text{if } \#V_j(c) < \#V_k(c) \end{cases} \quad (2) \\ \text{and } w_A(c) &= 1, w_B(c) = 0 \text{ if } \#V_A(c) = \#V_B(c). \end{aligned}$$

Our equilibrium concept in this paper is Nash equilibrium. Given a distribution of voters, a profile of campaign strategies  $c^* \in C$  is a (Nash) **equilibrium** if, for all political party  $j \neq k$  and all  $c'_j \in C_j$ ,  $w_j(c_j^*, c_k^*) \geq w_j(c'_j, c_k^*)$ .

## 3 Equilibrium analysis

To simplify the subsequent analysis it is convenient to focus on the case of two political issues, i.e.,  $M = \{1, 2\}$ . Note that if the political positions of both parties on an issue  $r \neq s$  are identical, then  $u_i(j, c) \geq u_i(k, c)$  if and only if  $-[x_{js} - \pi_{is}]^2 \geq -[x_{ks} - \pi_{is}]^2$ . This implies that issue  $r$  is “innocuous”, and that the same party will win the elections for all profile of campaign strategies. From now on we will assume that the political positions of the parties are different on both issues:  $x_{A1} \neq x_{B1}$  and  $x_{A2} \neq x_{B2}$ .

The utility function of each voter  $i$  can be rewritten as:

$$u_i(j, c) = -T(c) [x_{j1} - \pi_{i1}]^2 - [x_{j2} - \pi_{i2}]^2 \quad (3)$$

where  $T(c) = \frac{\alpha_1(c_1)}{\alpha_2(c_2)}$  can be interpreted as the relative intensity of voters' preferences over issue 1 when the profile of campaign strategies is  $c$ . Thus,

the grater  $T(c)$ , the more relevant is issue 1 compared to issue 2 in voters' preferences.

From (3), voter  $i$  is indifferent between the two parties (and then he abstains from voting) when his ideal political position satisfies the following condition:

$$\pi_{i2} = \frac{T(c) [x_{A1}^2 - x_{B1}^2] + [x_{A2}^2 - x_{B2}^2]}{2 [x_{A2} - x_{B2}]} - \frac{T(c) [x_{A1} - x_{B1}]}{[x_{A2} - x_{B2}]} \pi_{i1}. \quad (4)$$

Equation (4) allows us to distinguish between those voters that vote for party  $A$  and those voters that vote for party  $B$ . Figure 2 shows an example of that. In a slight abuse of notation, we use  $T(c)$  to denote the line defined by Equation (4). Any voter whose ideal political position is situated on this line is indifferent between the two parties. If the ideal political position of a voter is situated to the left of that line, he votes for party  $A$ . Similarly, if the ideal political position of a voter is situated to the right of that line, he votes for party  $B$ .

[INSERT FIGURE 2]

Note that the available campaign funds define some minimum and maximum values for  $T(c)$ : since each party  $j$  can expend at most  $\bar{c}_j$ , we have  $\frac{\alpha_1(0)}{\alpha_2(\bar{c}_A + \bar{c}_B)} \leq T(c) \leq \frac{\alpha_1(\bar{c}_A + \bar{c}_B)}{\alpha_2(0)}$  for all  $c \in C$ . We denote  $T_{\min} = \frac{\alpha_1(0)}{\alpha_2(\bar{c}_A + \bar{c}_B)}$  and  $T_{\max} = \frac{\alpha_1(\bar{c}_A + \bar{c}_B)}{\alpha_2(0)}$  to the minimum and maximum values of  $T(c)$ . These values are the key to know the subgroup of voters that may change his vote according to the specific profile of campaign strategies.

Consider, for example, the situation depicted in Figure 3. Again, we abuse of notation and use  $T_{\min}$  (resp.  $T_{\max}$ ) to denote the line defined by Expression (4) when  $T(c) = T_{\min}$  (resp.  $T(c) = T_{\max}$ ). In this example, any voter  $i$  whose ideal political position is situated below the lines  $T_{\min}$  and  $T_{\max}$  is such that  $u_i(A, c) > u_i(B, c)$  for all  $c \in C$ , and then he always votes for party  $A$ , no matter what the profile of campaign strategies is. Similarly, any voter whose ideal political position is situated above the lines  $T_{\min}$  and  $T_{\max}$  is such that  $u_i(B, c) > u_i(A, c)$  for all  $c \in C$ , and then he always votes for party  $B$ . We call these voters partisan voters. Formally, the **set of partisan voters of party  $j$**  (where  $j \neq k$ ) is given by  $P_j = \{i \in N : u_i(j, c) > u_i(k, c) \text{ for all } c \in C\}$ .

Note that  $\frac{\partial T_{\min}}{\partial(\bar{e}_A + \bar{e}_B)} < 0$  and  $\frac{\partial T_{\max}}{\partial(\bar{e}_A + \bar{e}_B)} > 0$ , and then, the greater the campaign funds are, the smaller the set of partisan voters is.

Any voter situated between the lines  $T_{\min}$  and  $T_{\max}$  is such that  $u_i(A, c) > u_i(B, c)$  for some  $c \in C$  and  $u_i(B, c') > u_i(A, c')$  for some  $c' \in C$ , and then his vote will depend on the particular profile of campaign strategies. We call these voters issue voters. Formally, given any profile of campaign strategies  $c \in C$ , the **set of issue voters of party j under c** is given by  $I_j(c) = V_j(c) \setminus P_j$ . The campaign expenditure on some particular issues can move the vote of an issue voter towards the party that best fits his preferences on these issues.<sup>6</sup>

[INSERT FIGURE 3]

As we have seen, voters' preferences over political parties depend on the relative importance that they assign to issue 1, which itself may vary within the range  $[T_{\min}, T_{\max}]$  depending on parties' campaign strategies  $c \in C$ . Then, given a distribution of voters, we can partition the interval  $[T_{\min}, T_{\max}]$  into different subintervals according to the party that wins the elections. We call it the **winning partition** of  $[T_{\min}, T_{\max}]$ .

The example illustrated in Figure 4 may clarify this concept. There are five voters represented by their ideal political positions  $\pi_i \in [0, 1]^2$ . The winning partition of  $[T_{\min}, T_{\max}]$  consists of four subintervals:  $[T_{\min}, T_1)$ ,  $[T_1, T_2]$ ,  $(T_2, T_3)$ , and  $[T_3, T_{\max}]$ . To see this note that:

- (1) For all  $T(c) \in [T_{\min}, T_1)$  the winning party is  $B$  (voters 1 and 4 vote for party  $A$  and voters 2, 3, and 5 vote for party  $B$ ).
- (2) For all  $T(c) \in [T_1, T_2]$  the winning party is  $A$  (in particular, if  $T(c) \in (T_1, T_2)$ , voters 1, 2, and 4 vote for party  $A$  and voters 3 and 5 vote for party  $B$ ; if  $T(c) = T_1$ , voters 1 and 4 vote for party  $A$ , voters 3 and 5 vote for party  $B$ , and voter 2 abstains; if  $T(c) = T_2$ , voters 1 and 2 vote for party  $A$ , voters 3 and 5 vote for party  $B$ , and voter 4 abstains).
- (3) For all  $T(c) \in (T_2, T_3)$  the winning party is  $B$  (voters 1 and 2 vote for party  $A$  and voters 3, 4, and 5 vote for party  $B$ ).

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<sup>6</sup>Some authors assume that the set of voters influenced by the campaign expenditure is a fixed fraction of uninformed voters (see, e.g., Baron, 1994, and Grossman and Helpman, 1996). In our model, however, it is the lack of partisan identification which makes some voters vulnerable to campaign activities.

(4) For all  $T(c) \in [T_3, T_{\max}]$  the winning party is  $A$  (in particular, if  $T(c) \in (T_3, T_{\max}]$ , voters 1, 2, and 3 vote for party  $A$  and voters 4 and 5 vote for party  $B$ ; if  $T(c) = T_3$ , voters 1 and 2 vote for party  $A$ , voters 4 and 5 vote for party  $B$ , and voter 3 abstains).

[INSERT FIGURE 4]

As we next show, a necessary (and sufficient) condition for equilibrium existence is that one of the parties has a weakly dominant strategy that ensures its victory. This implies that the equilibrium, if exists, is not unique, and that the same party wins the elections in any of these equilibria.

**Lemma 1** *Given a distribution of voters, let  $c^* \in C$  be an equilibrium where party  $j$  wins the elections. Then,*

- i)  $c_j^*$  is a weakly dominant strategy that ensures the victory of party  $j$ ,*
- ii) the equilibrium is not unique, and*
- iii) party  $j$  wins the elections in any other equilibrium.*

**Proof.** Let  $(c_A^*, c_B^*) \in C$  be an equilibrium where party  $A$  wins the elections. Then, from the parties' utility functions it follows that, for all  $c'_B \in C_B$ ,  $0 = w_B(c_A^*, c_B^*) \geq w_B(c_A^*, c'_B)$ . Hence, for all  $c'_B \in C_B$ ,  $w_B(c_A^*, c'_B) = 0$  and  $w_A(c_A^*, c'_B) = 1$ . This implies that  $c_A^*$  is a weakly dominant strategy of party  $A$  that ensures its victory, and hence there is no other equilibrium where party  $A$  does not win the elections. Moreover, for all  $c'_B \in C_B$ ,  $(c_A^*, c'_B)$  is also an equilibrium. The case where party  $B$  wins the elections is analogous. ■

While the above result is valid for any number of political issues, when we focus on two political issues the conditions of Lemma 1 can be interpreted in terms of  $T(c)$ . Thus, the next lemma states that a profile of campaign strategies  $c^* \in C$  is an equilibrium if and only if the party that wins the elections for  $T(c^*)$  still wins for any  $T(c)$  which is attainable by means of a unilateral deviation of the other party.

**Lemma 2** *A profile of campaign strategies  $c^* \in C$  where party  $j \neq k$  wins the elections is an equilibrium if and only if party  $j$  wins the elections for all  $T(c) \in \left[ \frac{\alpha_1(c_{j1}^*)}{\alpha_2(c_{j2}^* + \bar{c}_k)}, \frac{\alpha_1(c_{j1}^* + \bar{c}_k)}{\alpha_2(c_{j2}^*)} \right]$ .*<sup>7</sup>

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<sup>7</sup>Given that the parties do not need to spend all their campaign funds and that the influence functions are continuous, party  $k$  can make  $T(c)$  to take any value in  $\left[ \frac{\alpha_1(c_{j1}^*)}{\alpha_2(c_{j2}^* + \bar{c}_k)}, \frac{\alpha_1(c_{j1}^* + \bar{c}_k)}{\alpha_2(c_{j2}^*)} \right]$  by means of some unilateral deviation from  $c^*$ .

The proof of this result is direct and we omit it in the interest of brevity. Note that Lemma 2 can be interpreted in terms of the winning partition of  $[T_{\min}, T_{\max}]$ : given that party  $j$  wins the elections for  $T(c^*)$ ,  $c^* \in C$  is an equilibrium if and only if  $\left[ \frac{\alpha_1(c_{j1}^*)}{\alpha_2(c_{j2}^* + \bar{c}_k)}, \frac{\alpha_1(c_{j1}^* + \bar{c}_k)}{\alpha_2(c_{j2}^*)} \right]$  is included in the same subinterval of the winning partition of  $[T_{\min}, T_{\max}]$  than  $T(c^*)$ .

In Figure 5 we show an equilibrium situation (top graph) and a non-equilibrium situation (bottom graph). The winning partition of  $[T_{\min}, T_{\max}]$  is the same one we obtained in the example illustrated in Figure 4. Consider first the profile of strategies  $c^*$ . Since  $T(c^*)$  is in the subinterval  $[T_1, T_2]$ , party  $A$  will win the elections under  $c^*$ . Moreover, since  $\left[ \frac{\alpha_1(c_{A1}^*)}{\alpha_2(c_{A2}^* + \bar{c}_B)}, \frac{\alpha_1(c_{A1}^* + \bar{c}_B)}{\alpha_2(c_{A2}^*)} \right]$  is also included in  $[T_1, T_2]$ , party  $A$  will win the elections for any unilateral deviation of party  $B$ . Therefore,  $c^*$  is an equilibrium. Consider now the profile of strategies  $c'$  (bottom graph). Since  $T(c')$  is in the subinterval  $[T_3, T_{\max}]$ , party  $A$  will win the elections under  $c'$ . Nevertheless, since  $\left[ \frac{\alpha_1(c'_{A1})}{\alpha_2(c'_{A2} + \bar{c}_B)}, \frac{\alpha_1(c'_{A1} + \bar{c}_B)}{\alpha_2(c'_{A2})} \right]$  is not included in  $[T_3, T_{\max}]$ ,  $c'$  is not an equilibrium. In particular, we find that party  $B$  could move  $T(c')$  towards the subinterval  $(T_2, T_3)$  and win the elections.

[INSERT FIGURE 5]

The following lemma provides a simple procedure to check which party wins the elections in the case that an equilibrium exists.

**Lemma 3** *Suppose that  $A$  has at least as much money as party  $B$  (i.e.,  $\bar{c}_A \geq \bar{c}_B$ ). If there exists an equilibrium where party  $B$  wins the elections, then party  $B$  must win the elections for all  $T(c) \in \left[ \frac{\alpha_1(\bar{c}_B)}{\alpha_2(\bar{c}_A)}, \frac{\alpha_1(\bar{c}_A)}{\alpha_2(\bar{c}_B)} \right]$ . If there exists an equilibrium where party  $A$  wins the election, then party  $A$  must win the election for at least one  $T(c) \in \left[ \frac{\alpha_1(\bar{c}_B)}{\alpha_2(\bar{c}_A)}, \frac{\alpha_1(\bar{c}_A)}{\alpha_2(\bar{c}_B)} \right]$ .*

**Proof.** By Lemma 2, if  $c^* \in C$  is an equilibrium where party  $B$  wins the elections, then party  $B$  wins for all  $T(c) \in \left[ \frac{\alpha_1(c_{B1}^*)}{\alpha_2(c_{B2}^* + \bar{c}_A)}, \frac{\alpha_1(c_{B1}^* + \bar{c}_A)}{\alpha_2(c_{B2}^*)} \right]$ . Since  $\bar{c}_A \geq \bar{c}_B$  and  $\alpha_1(\cdot), \alpha_2(\cdot)$  are strictly increasing functions,  $\frac{\alpha_1(\bar{c}_B)}{\alpha_2(\bar{c}_A)} \leq \frac{\alpha_1(\bar{c}_A)}{\alpha_2(\bar{c}_B)}$  and  $\left[ \frac{\alpha_1(\bar{c}_B)}{\alpha_2(\bar{c}_A)}, \frac{\alpha_1(\bar{c}_A)}{\alpha_2(\bar{c}_B)} \right] \subseteq \left[ \frac{\alpha_1(c_{B1}^*)}{\alpha_2(c_{B2}^* + \bar{c}_A)}, \frac{\alpha_1(c_{B1}^* + \bar{c}_A)}{\alpha_2(c_{B2}^*)} \right]$ . If  $c^* \in C$  is an equilibrium where

party  $A$  wins the elections, then party  $A$  wins for all  $T(c) \in \left[ \frac{\alpha_1(c_{A1}^*)}{\alpha_2(c_{A2}^* + \bar{c}_B)}, \frac{\alpha_1(c_{A1}^* + \bar{c}_B)}{\alpha_2(c_{A2}^*)} \right]$ . Since  $\frac{\alpha_1(\bar{c}_A)}{\alpha_2(\bar{c}_B)} \geq \frac{\alpha_1(c_{A1}^*)}{\alpha_2(c_{A2}^* + \bar{c}_B)}$  and  $\frac{\alpha_1(\bar{c}_B)}{\alpha_2(\bar{c}_A)} \leq \frac{\alpha_1(c_{A1}^* + \bar{c}_B)}{\alpha_2(c_{A2}^*)}$ , then  $\left[ \frac{\alpha_1(\bar{c}_B)}{\alpha_2(\bar{c}_A)}, \frac{\alpha_1(\bar{c}_A)}{\alpha_2(\bar{c}_B)} \right] \cap \left[ \frac{\alpha_1(c_{A1}^*)}{\alpha_2(c_{A2}^* + \bar{c}_B)}, \frac{\alpha_1(c_{A1}^* + \bar{c}_B)}{\alpha_2(c_{A2}^*)} \right] \neq \emptyset$ . ■

There are certain scenarios where existence of equilibrium can be guaranteed. Our next lemma describes these scenarios in terms of the number of subintervals of the winning partition  $[T_{\min}, T_{\max}]$ .

**Theorem 1** *If the winning partition of  $[T_{\min}, T_{\max}]$  consists of one or two subintervals, then equilibrium exists.*

**Proof.** Suppose that the winning partition of  $[T_{\min}, T_{\max}]$  has only one subinterval where party  $j$  wins the elections. Then by Lemma 1, any profile of campaign strategies constitutes an equilibrium where party  $j$  wins the elections.

Suppose, without loss of generality, that there exists some critical level  $T_1 \in [T_{\min}, T_{\max}]$  such that party  $A$  wins the elections for all  $T(c) \in [T_{\min}, T_1]$ , while party  $B$  wins the elections for all  $T(c) \in (T_1, T_{\max}]$ . If  $\frac{\alpha_1(\bar{c}_B)}{\alpha_2(\bar{c}_A)} \leq T_1$ , then by Lemma 2,  $c_A^* = (0, \bar{c}_A) \in C_A$  is a strategy that ensures the victory of party  $A$ . If  $\frac{\alpha_1(\bar{c}_B)}{\alpha_2(\bar{c}_A)} > T_1$ , then by Lemma 2,  $c_B^* = (\bar{c}_B, 0) \in C_B$  is a strategy that ensures the victory of party  $B$ . Therefore, an equilibrium exists. ■

An example where the winning partition has only one subinterval is given when the number of partisan voters of some party  $j$  is greater than  $\frac{n}{2}$ . The type of equilibrium that arises when there are two subintervals is such that each party has an advantage on one of the political issues but one of the parties can ensure its victory by means of expending all its campaign funds on its advantageous issue.

Unfortunately, equilibrium existence problems may arise when the winning partition of  $[T_{\min}, T_{\max}]$  has three or more subintervals.

**Theorem 2** *If the winning partition of  $[T_{\min}, T_{\max}]$  consists of three or more subintervals, then an equilibrium can fail to exist.*

**Proof.** Consider an example where the influence functions are:

Issue 1	Issue 2
$\alpha_1(c_1) = 1 + \sqrt{c_1}$	$\alpha_2(c_2) = 2 + \sqrt{c_2}$

The parties' political positions and campaign funds are described bellow.

Party $A$	Party $B$
$x_{A1} = 0.3$	$x_{B1} = 0.7$
$x_{A2} = 0.3$	$x_{B2} = 0.7$
$\bar{c}_A = 16$	$\bar{c}_B = 9$

The set of voters is composed of five individuals whose ideal political positions are the following:

$N$				
Voter 1	Voter 2	Voter 3	Voter 4	Voter 5
$\pi_{11} = 0.1$	$\pi_{21} = 0.1$	$\pi_{31} = 0.4$	$\pi_{41} = 0.8$	$\pi_{51} = 0.9$
$\pi_{12} = 0.2$	$\pi_{22} = 0.7$	$\pi_{32} = 0.9$	$\pi_{42} = 0.9$	$\pi_{52} = 0.2$

Note that the relative intensity of voters' preferences over issue 1 may vary between  $T_{\min} = \frac{\alpha_1(0)}{\alpha_2(\bar{c}_A + \bar{c}_B)} = \frac{1}{7}$  and  $T_{\max} = \frac{\alpha_1(\bar{c}_A + \bar{c}_B)}{\alpha_2(0)} = 3$ , depending on the campaign strategies  $c \in C$ . Given the preferences of the voters over political parties we have that:

- (1) Voter 1 votes for party  $A$  for all  $T(c) \in [T_{\min}, T_{\max}]$ .
- (2) Voter 2 votes for party  $B$  when  $T(c) \in [T_{\min}, 0.5)$ , votes for party  $A$  when  $T(c) \in (0.5, T_{\max}]$ , and abstains when  $T(c) = 0.5$ .
- (3) Voter 3 votes for party  $B$  for all  $T(c) \in [T_{\min}, T_{\max}]$ .
- (4) Voter 4 votes for party  $B$  for all  $T(c) \in [T_{\min}, T_{\max}]$ .
- (5) Voter 5 votes for party  $A$  when  $T(c) \in [T_{\min}, 0.75)$ , votes for party  $B$  when  $T(c) \in (0.75, T_{\max}]$ , and abstains when  $T(c) = 0.75$ .

Therefore, the winning partition of  $[T_{\min}, T_{\max}]$  consists of the following three subintervals.

Winning partition of $[T_{\min}, T_{\max}]$		
$[T_{\min}, 0.5)$	$[0.5, 0.75]$	$(0.75, T_{\max}]$
Party $B$ wins	Party $A$ wins	Party $B$ wins

Since  $\left[ \frac{\alpha_1(\bar{c}_B)}{\alpha_2(\bar{c}_A)}, \frac{\alpha_1(\bar{c}_A)}{\alpha_2(\bar{c}_B)} \right] = \left[ \frac{2}{3}, 1 \right]$  is not included in any subinterval of the winning partition where party  $B$  wins the elections, we know from Lemma 3 that there is no equilibrium where party  $B$  wins the elections. Moreover, there is no equilibrium where party  $A$  wins the elections. To see this note that: (1) for any  $c_A \in C_A$  with  $c_{A2} < \frac{100}{9}$ , there is some  $c_B \in C_B$  such that  $T(c) \in$

$(0.75, T_{\max}]$  (and then party  $B$  wins the elections), and (2) for any  $c_A \in C_A$  with  $c_{A2} > 11$ , there is some  $c_B \in C_B$  such that  $T(c) \in [T_{\min}, 0.5)$  (and then party  $B$  wins the elections). The extension to four or more subintervals follows from including some additional issue voters.<sup>8</sup> ■

## 4 Explaining surprising results

This section analyzes to what extent the following three factors can guarantee the victory of a political party: i) winning the elections if there is no campaign expenditure, ii) having advantage on some political issues, and iii) having more campaign funds than the opponent. We next describe in more detail each of these factors.

The first factor is used as a benchmark. It describes the case where there is no campaign expenditure. We write  $c = 0$  to denote this situation. Similarly, we will denote  $T_0 = \frac{\alpha_1(0)}{\alpha_2(0)}$  to the relative intensity of voters' preferences over issue 1 when parties do not spend money in the electoral campaign (note that  $T_{\min} < T_0 < T_{\max}$ ). The party that wins the elections in this case will be identified as the **ex-ante winner**.

The second factor crucially depends on the voters' ideal political positions on each particular issue. We say that party  $j$  has an advantage on issue  $r$  when party  $j$  would win some hypothetical elections where individuals only care about that particular issue. Formally, **party  $j$  has an advantage on issue  $r$**  when  $\#\{i \in N : [x_{jr} - \pi_{ir}]^2 > [x_{kr} - \pi_{ir}]^2\} > \#\{i \in N : [x_{jr} - \pi_{ir}]^2 < [x_{kr} - \pi_{ir}]^2\}$  where  $j \neq k$ .<sup>9</sup> In Figure 4, for instance, party  $A$  has an advantage on issue 1 whereas party  $B$  has an advantage on issue 2.

The third factor is the campaign funds difference between parties. Having more campaign funds than the opponent shall increase the possibilities that one party has to influence voters' preferences.

Surprisingly, the following result shows that none of the above mentioned factors guarantees the victory of a political party.

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<sup>8</sup>We just consider pure strategy equilibria. The proposed electoral game is a zero sum game where the set of strategies is infinite and uncountable. To guarantee existence of mixed strategy equilibria the parties' objective function should be continuous, which, it is not the case in our model. See Riker and Ordeshook (1973, p. 216).

<sup>9</sup>We suppose that, if  $[x_{Ar} - \pi_{ir}]^2 = [x_{Br} - \pi_{ir}]^2$ , then voter  $i$  abstains from voting in the hypothetical one-issue elections.



**Theorem 3** *There are some distributions of voters for which the ex-ante winner loses the elections, even when it has an advantage on every single political issue and it has more campaign funds than its opponent.*

**Proof.** Consider the example illustrated in Figure 6. The influence functions are the following:

Issue 1	Issue 2
$\alpha_1(c_1) = 1 + \sqrt{c_1}$	$\alpha_2(c_2) = 1 + 2\sqrt{c_2}$

The parties have the following political positions and campaign funds:

Party A	Party B
$x_{A1} = 0.3$	$x_{B1} = 0.7$
$x_{A2} = 0.3$	$x_{B2} = 0.7$
$\bar{c}_A = 16$	$\bar{c}_B = 9$

The set of voters is partitioned in the following four subsets:

$N$			
$N_1 : 9 \text{ voters}$	$N_2 : 3 \text{ voters}$	$N_3 : 5 \text{ voters}$	$N_4 : 4 \text{ voters}$
$\pi_{i1} = 0.1$	$\pi_{i1} = 0.2$	$\pi_{i1} = 0.9$	$\pi_{i1} = 0.8$
$\pi_{i2} = 0.1$	$\pi_{i2} = 0.77$	$\pi_{i2} = 0.9$	$\pi_{i2} = 0.44$

The relative intensity of voters' preferences over issue 1 may vary between  $T_{\min} = \frac{\alpha_1(0)}{\alpha_2(\bar{c}_A + \bar{c}_B)} = \frac{1}{11}$  and  $T_{\max} = \frac{\alpha_1(\bar{c}_A + \bar{c}_B)}{\alpha_2(0)} = 6$ , depending on the campaign strategies  $c \in C$ . Given the preferences of voters over political parties we have that:

- (1) Voters in  $N_1$  vote for party A for all  $T(c) \in [T_{\min}, T_{\max}]$ .
- (2) Voters in  $N_2$  vote for party B when  $T(c) \in [T_{\min}, 0.9)$ , and vote for party A when  $T(c) \in (0.9, T_{\max}]$ .
- (3) Voters in  $N_3$  vote for party B for all  $T(c) \in [T_{\min}, T_{\max}]$ .
- (4) Voters in  $N_4$  vote for party A when  $T(c) \in [T_{\min}, 0.2)$ , and vote for party B when  $T(c) \in (0.2, T_{\max}]$ .

Therefore, the winning partition of  $[T_{\min}, T_{\max}]$  is the following:

Winning partition of $[T_{\min}, T_{\max}]$		
$[T_{\min}, 0.2]$	$(0.2, 0.9)$	$[0.9, T_{\max}]$
Party A wins	Party B wins	Party A wins

Note that  $T_0 = \frac{\alpha_1(0)}{\alpha_2(0)} = 1$ , and therefore party  $A$  is ex-ante winner. Moreover, as we next show, party  $A$  wins any hypothetical one-issue elections. Party  $A$ 's political position on issue 1 is supported by  $\frac{12}{21}$  of the population (voters in  $N_1$  and  $N_2$  strictly prefer  $A$ 's political position on issue 1 rather than  $B$ 's political position on that issue). Similarly, party  $A$ 's political position on issue 2 is supported by  $\frac{13}{21}$  of the population (voters  $N_1$  and  $N_4$  strictly prefer  $A$ 's political position on issue 2 rather than  $B$ 's political position on that issue). Nevertheless, party  $B$  wins the elections in equilibrium. To see this note that (for example)  $c_B^* = (c_{B1}^*, c_{B2}^*) = (2, 7)$  is a weakly dominant strategy for party  $B$  that ensures its victory, since applying Lemma 2,  $\left[ \frac{\alpha_1(c_{B1}^*)}{\alpha_2(c_{B2}^* + \bar{c}_A)}, \frac{\alpha_1(\bar{c}_A + c_{B1}^*)}{\alpha_2(c_{B2}^*)} \right] = \left[ \frac{1+\sqrt{2}}{1+2\sqrt{23}}, \frac{1+\sqrt{18}}{1+2\sqrt{7}} \right] \subset (0.2, 0.9)$ . ■

[INSERT FIGURE 6]

In the example of Theorem 3, the set of issue voters is crucial to win the elections. As we have shown, there is a subinterval of the winning partition where party  $B$  can win the elections. Party  $B$  can reach that subinterval by slightly influencing preferences towards issue 2. The idea is that by doing so, party  $B$  captures the voters in  $N_2$  without losing the voters in  $N_4$  (see Figure 6). If instead party  $B$  spends all its campaign funds on issue 2, then it is party  $A$  who wins.<sup>10</sup> It is important to note that the proposed influence function on issue 2 is more sensitive than the influence function on issue 1. Thus, although party  $A$  may try to compensate the campaign strategy of party  $B$ , it is unable to reach the ex-ante elections result. Technically the reason for this is that  $\frac{\alpha_1(\bar{c}_A + c_{B1}^*)}{\alpha_2(c_{B2}^*)} < T_0$ .

Thus, in spite of having less campaign funds and no advantage on any issue, the ex-ante loser may have some chances of winning the elections. There are, however, some limits on the campaign funds difference for the ex-ante loser to win the elections. Our next remark establishes a necessary connection between the influence functions and the campaign funds difference in order to have an equilibrium where the ex-ante loser wins the election despite of having less campaign funds than its opponent.

<sup>10</sup>Note that  $\frac{\alpha_1(0)}{\alpha_2(\bar{c}_B)} = \frac{1}{7} \in [T_{\min}, 0.2]$ , where party  $A$  wins.

**Remark 1** Suppose that party  $A$  is the ex-ante winner and that  $\bar{c}_A \geq \bar{c}_B$ . Suppose that there exists an equilibrium where party  $B$  wins the elections. Then, there is some issue  $r \neq s$  such that  $\frac{\alpha_r(\bar{c}_B)}{\alpha_r(0)} > \frac{\alpha_s(\bar{c}_A)}{\alpha_s(0)}$ .

**Proof.** Since there exists an equilibrium where party  $B$  wins the elections, from Lemma 2 we know that party  $B$  wins the elections for all  $T(c) \in \left[ \frac{\alpha_1(\bar{c}_B)}{\alpha_2(\bar{c}_A)}, \frac{\alpha_1(\bar{c}_A)}{\alpha_2(\bar{c}_B)} \right]$ . Therefore, either  $T_0 < \frac{\alpha_1(\bar{c}_B)}{\alpha_2(\bar{c}_A)}$  or  $T_0 > \frac{\alpha_1(\bar{c}_A)}{\alpha_2(\bar{c}_B)}$  (otherwise party  $A$  is not the ex-ante winner). In the first case we have  $\frac{\alpha_1(\bar{c}_B)}{\alpha_1(0)} > \frac{\alpha_2(\bar{c}_A)}{\alpha_2(0)}$ , while in the second case we have  $\frac{\alpha_2(\bar{c}_B)}{\alpha_2(0)} > \frac{\alpha_1(\bar{c}_A)}{\alpha_1(0)}$ .<sup>11</sup> ■

Note that, if  $\bar{c}_A \geq \bar{c}_B$  and  $\frac{\alpha_r(\bar{c}_B)}{\alpha_r(0)} > \frac{\alpha_s(\bar{c}_A)}{\alpha_s(0)}$ , then  $\alpha_r$  is more sensitive to an increase in the campaign expenditure than  $\alpha_s$  (the rate of growth of  $\alpha_r$  when  $c_r$  changes from 0 to  $\bar{c}_B$  is greater than the rate of growth of  $\alpha_s$  when  $c_s$  changes from 0 to  $\bar{c}_A$ ). The intuition is that, if  $\bar{c}_A \geq \bar{c}_B$  and both issues were equally sensitive to campaign expenditure, party  $A$  could always compensate the strategy of party  $B$  so that  $T(c) = T_0$ , and win the elections.

There are also some limits on the extent of the hypothetical one-issue defeats for having a chance of winning the elections. Whereas having an advantage on every single issue does not necessarily guarantee winning the elections, the victory is ensured if the one-issue advantages are large enough.

**Remark 2** Let  $n_{jr}$  be the number of voters that would vote for party  $j$  if they were only interested on issue  $r$ . If  $n_{j1} + n_{j2} > \frac{3}{2}n$ , then party  $j$  wins the elections, no matter how much campaign funds it has.<sup>12</sup>

**Proof.** (Figure 7) For all party  $j \neq k$  and all issue  $r \in M$ , let  $n_{jr} \geq 0$  be the number of voters that prefer the policy position of party  $j$  on issue  $r$ , i.e.,  $n_{jr} = \#\{i \in N: [x_{jr} - \pi_{ir}]^2 < [x_{kr} - \pi_{ir}]^2\}$ . Similarly, for all pair of issues  $r \neq s$ , let  $n_{Ar,Bs} \geq 0$  be the number of voters that prefer the policy position of  $A$  on issue  $r$ , but on issue  $s$  they prefer the policy position of party  $B$ , i.e.,  $n_{Ar,Bs} = \#\{i \in N: [x_{Ar} - \pi_{ir}]^2 < [x_{Br} - \pi_{ir}]^2 \text{ and } [x_{As} - \pi_{is}]^2 > [x_{Bs} - \pi_{is}]^2\}$ . Finally, let  $n_j^* = \#\{i \in N: [x_{jr} - \pi_{ir}]^2 \leq [x_{kr} - \pi_{ir}]^2 \text{ for all}$

<sup>11</sup>Note that in the example of Theorem 3  $\frac{\alpha_2(\bar{c}_B)}{\alpha_2(0)} = 7 > 5 = \frac{\alpha_1(\bar{c}_A)}{\alpha_1(0)}$ , and that the equilibrium strategy proposed for party  $B$  is such that it spends most of its funds on issue 2.

<sup>12</sup>The fact that  $n_{k1} + n_{k2} < \frac{n}{2}$  does not necessarily imply that  $n_{j1} + n_{j2} > \frac{3}{2}n$ , since some voters could abstain in the hypothetical one-issue elections.

$r \in M$ , with strict inequality for some  $r \in M$ . Note that, since individuals take into account both issues simultaneously, the  $n_j^*$  voters will always vote for party  $j$  (i.e., the  $n_j^*$  voters are partisan voters of party  $j$ ). Suppose that, for some party  $j \neq k$ ,  $n_{j1} + n_{j2} > \frac{3}{2}n$ . Since  $n_{j1} \leq n_j^* + n_{j1,k2}$ ,  $n_{j2} \leq n_j^* + n_{j2,k1}$ , and  $n_j^* + n_{j1,k2} + n_{j2,k1} \leq n$ , we have  $\frac{n}{2} < n_j^*$  and therefore party  $j$  will always win the elections.<sup>13</sup> ■

[INSERT FIGURE 7]

Thus, we find that there is a relation between the results obtained on the hypothetical one-issue elections and the chances that a political party has of winning the elections. If the sum of votes that a party obtains in the hypothetical one-issue elections is greater than  $\frac{3}{2}n$ , then it can be guaranteed that there is no way of defeating this party in the elections. Note that a particular instance for that condition is given by obtaining more than  $\frac{3}{4}n$  votes on each of the hypothetical one-issue elections, which, without doubt, requires having a great advantage over the opponent.

## 5 Conclusion

This paper proposes a model through which political campaign expenditure affects the elections result. In doing so, we have focused on the role that the advertising campaign plays on the weight that voters assign to each political issue. When none of the political parties has a majority of the electorate as partisan voters, the proposed electoral game becomes a competition between the two political parties to capture issue voters.

Although existence of equilibrium can be guaranteed for certain distributions of voters, equilibrium existence problems may arise when the elections result is more vulnerable to the intensity of voters' preferences over political issues.

We provide a surprising result which illustrates that a political party which, i) is the ex-ante winner, ii) has an advantage on all political issues, and iii) has more campaign funds than the opponent, can still be defeated by its

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<sup>13</sup>This result can be generalized to the case of  $m \geq 2$  issues: if  $\sum_{r=1}^m n_{jr} > \frac{2m-1}{2}n$ , then party  $j$  win the election, no matter how much campaign funds the parties have.

opponent. This result also illustrates that the strategy to capture issue voters does not need to be designed so that certain issues are strongly emphasized while other issues are omitted. As we show, for certain distribution of voters, slightly emphasizing some issues more than others may be the best strategy to capture issue voters. This result certainly contrasts the conventional wisdom of considering that political issues are such that the emphasis on an issue either benefits one and only one of the political parties, or it is innocuous for both political parties.

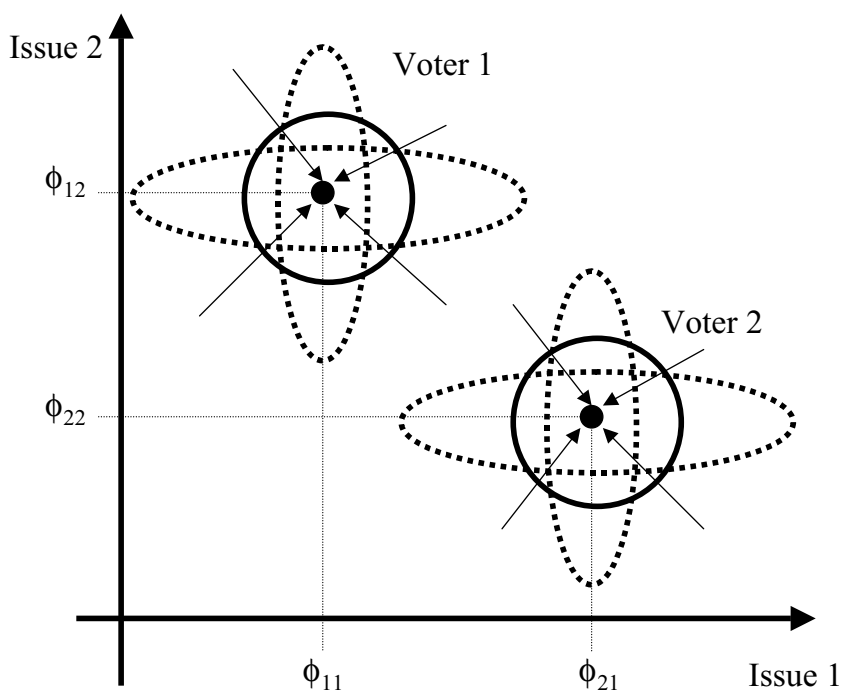
Voters' preferences can show certain asymmetry regarding how sensitive they are to the campaign expenditure on different issues. Thus, even if a political party has less campaign funds, it can have more impact on voters' preferences than its opponent. We show that this asymmetry is a necessary condition for the party with less funds to have any chance of winning the elections.

Finally, we show that a sufficient condition to win the elections consists of having a majority of  $\frac{3}{4}$  on each hypothetical one-issue elections. In this case, it does not matter whether or not the opponent has more campaign funds, since there is no way of defeating such political party.

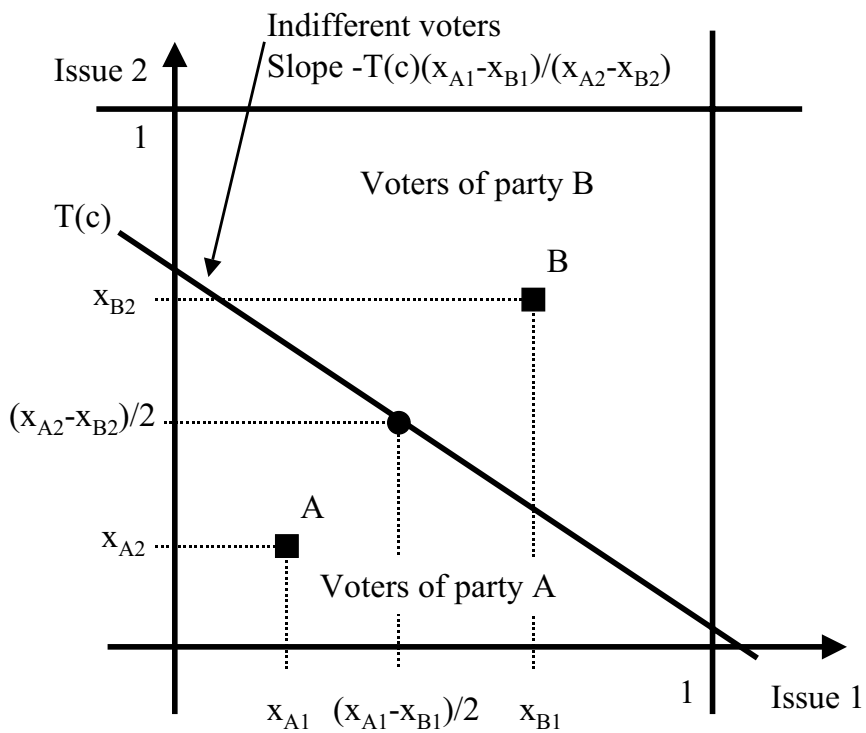
Extensions which account for more than two political parties, or which distinguish among groups of voters who weight issues differently (as for instance gender or race groups) are left for future research.

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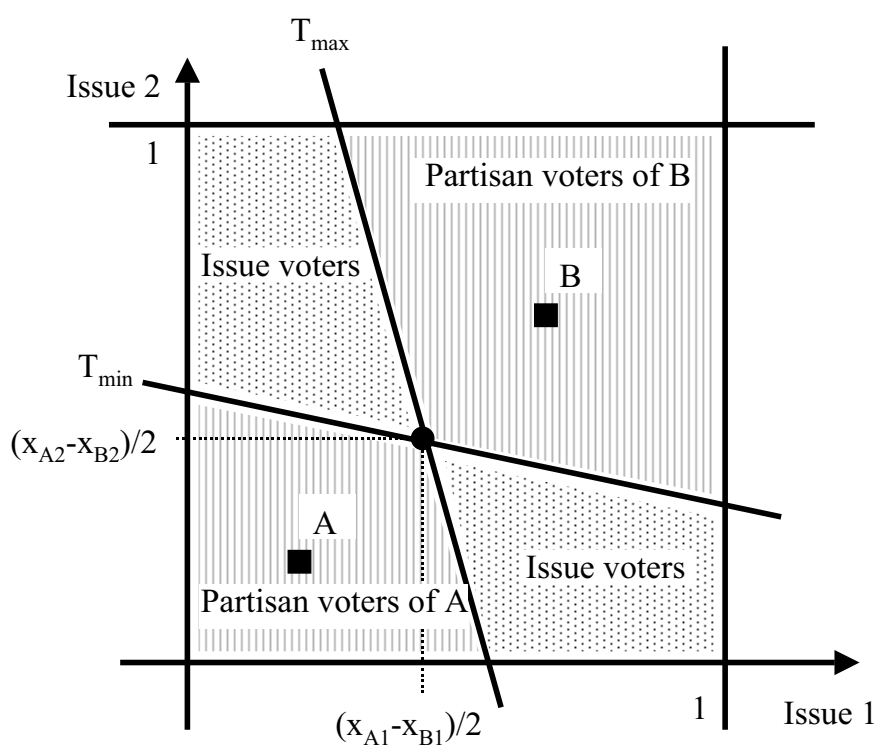


**Figure 1.** Example of voters' indifference curves.

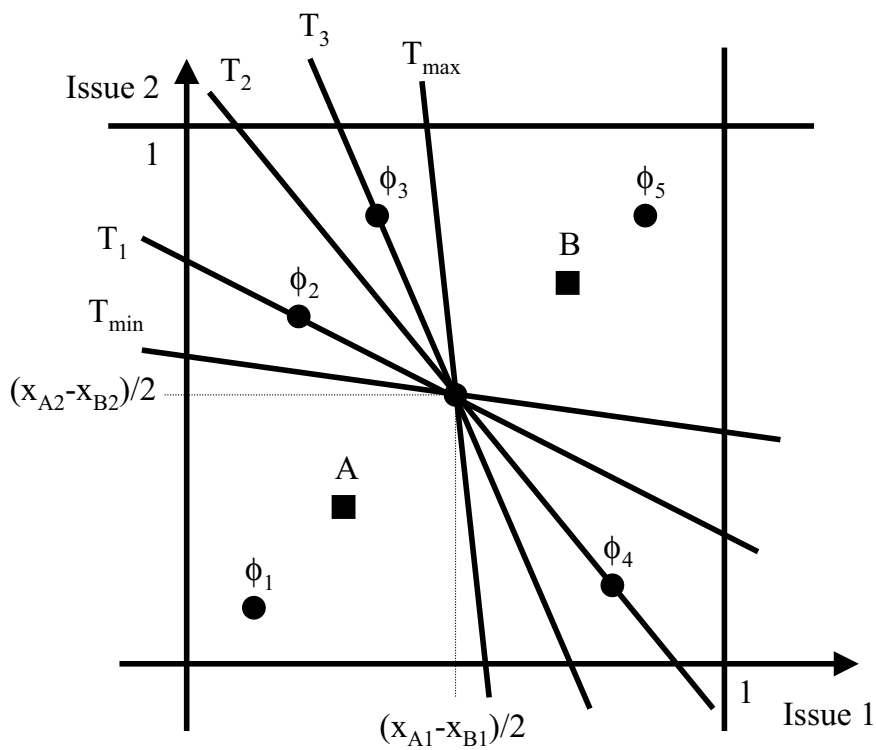


**Figure 2.** Example of voters of party A and party B.

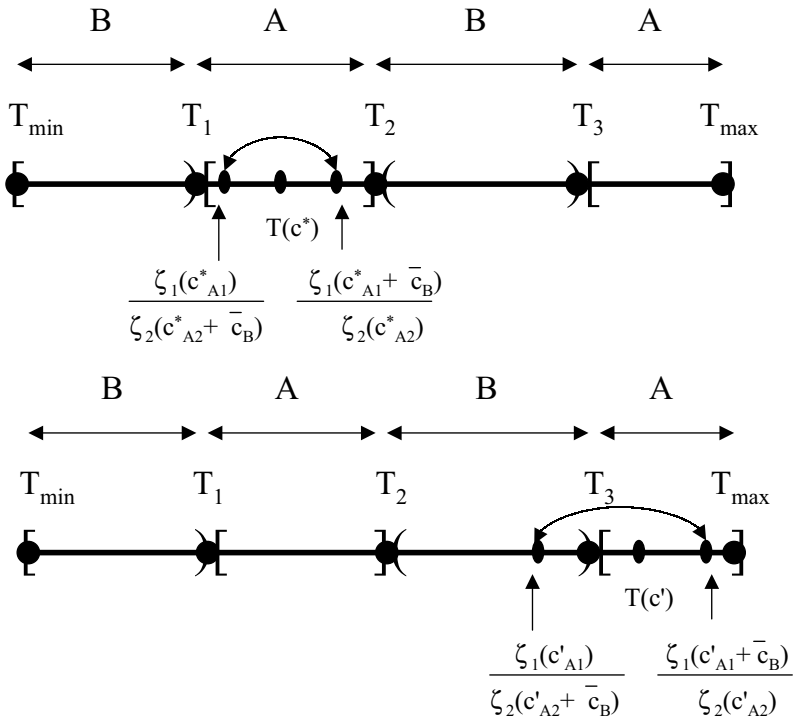




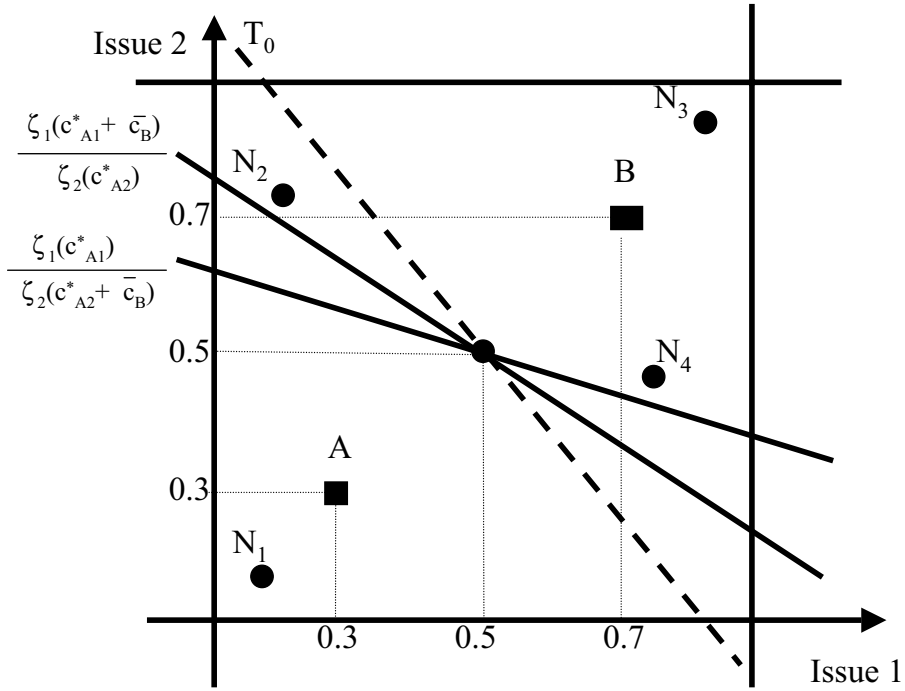
**Figure 3.** Example of partisan voters and issue voters.



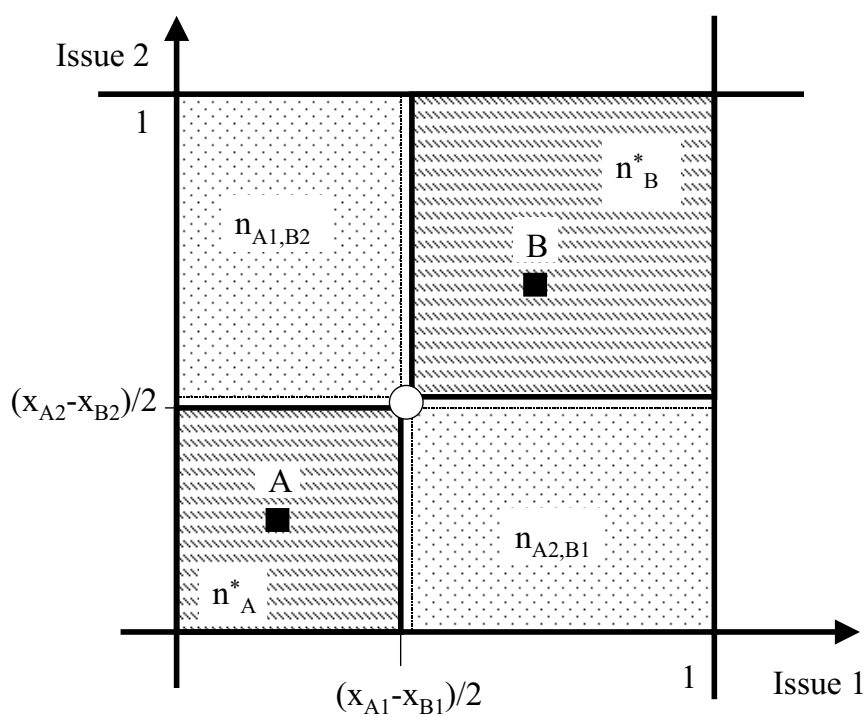
**Figure 4.** Computing the winning partition: an example.



**Figure 5.** Equilibrium and non-equilibrium situations in terms of the winning partition.



**Figure 6.** Example of Theorem 1.



**Figure 7.** Illustration of Remark 2.





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